Asset Allocation: Performance of Mean-Variance Strategy for Small Sample of Canadian Assets

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Abstract. *This project estimates performance of two out of sample Mean-Variance Strategy portfolios. Weights are rebalanced monthly. Five years rolling window portfolio uses stock return information of 5 preceding years (information set). Similarly recursive portfolio uses all information of preceding years to construct weights. Expected returns and covariance are assumed to be equal to mean of these parameters over available information set. Performance of these portfolios is compared with equal initial weights and every period equal weight portfolios. In order to do so we employed a data from November 1992 to November 2013 for 10 Canadian stocks and Treasury Bills. Ex post Sharpe ratios and cumulative returns indicated that Mean-Variance out of sample portfolios delivered lower results comparable to naïve portfolio strategies over last 15 years. We explain it by low correlation between expected and realized returns and hence by investing in wrong stocks in a wrong time. And we provide numerical and graphical evidence.*

Keywords: *asset, portfolio, return, stocks, strategy.*

1 Introduction

Investors prefer high return and low risks. When deciding on investment strategy, investor might try to minimize risk for a given rate of return or maximize expected return given some risk. Minimum-variance optimization strategy solves the aforementioned problem by producing optimal allocation weights of investor's wealth among different assets. Our objective in this paper is to assess the risk-adjusted performance of the minimum-variance investment strategies for equity investors relative to the performance of the naive 1/N portfolio.

Previous studies (Merton (1980), Nelson (1992)) identified that variance-covariance matrix estimates are relatively stable over time and characterized by lower estimation errors in comparison to returns estimation process. Other studies (Baker and Haugen (1991), Clarke et al. (2006), Chan et al. (1999), Jagannathan and Ma (2003), DeMiguel et al. (2009)) showed that minimum-variance portfolio out-of-sample performance is better compared to value weighted and tangency portfolios. In DeMiguel et al. (2009) study, the minimum-variance portfolio with constraints performed most favorably in terms of Sharpe ratio out of 14 models, but its performance relative to 1/N strategy is less clear.

Having estimated variance-covariance matrix from historical data, we adjusted portfolio weights and set a constraint that sum of weights should be equal to 1. For Mean-Variance portfolio we allow short sales. We chose to rebalance the portfolio every month, using different estimation windows: five-year rolling-estimation window and recursive method for estimation of the parameters. As expected return and expected covariance we take average return and covariance over information set available in the day of constructing/rebalancing weights.

We compare the out-of-sample performance of two portfolio models relative to that of the Equal weight portfolio. It is found that Equal weigh portfolio beats out of sample min.var. portfolios for selected 11 assets from 1998–2013. We explain it by the fact that taking average return over last 5 years or all available information set does not predict expected value for return. It is consistent with the literature. Also, we explain it by the possible problems in predicting covariance. Additional explanation comes from the fact that sample includes only 10 risky assets and some stocks, that had high weight, initially did very badly over the period and vice versa.

Contribution of this paper is in the evaluation of the dynamic minimum variance portfolio strategy on the most recent dataset of returns of some well-known Canadian companies. Another contribution follows from an example when very high transaction costs starting from the 2nd period can improve portfolio performance. We do not argue that it is always the case, but it is shown that for this set of assets, time and intended investing strategy it is a case. We argue that it can prevent backward looking investor from buying expensive stocks and selling cheap stocks.

The outline for the rest of the paper is as follows: The second section describes the dataset used in this empirical study. Section three reviews the theoretical foundation consisting of methodology on minimum-variance portfolio optimization, benchmark portfolio calculation and performance evaluation. The fourth section presents the empirical results on two minimum variance strategies (5 years rolling window and recursive) compared to equally-weighted portfolio. Then fifth section discusses possible impact of transaction costs. Finally, the sixth section concludes the findings of this paper.

2 Description of the data

Monthly historical prices were downloaded in Research Data Center from Thomson Reuters Data Stream (in CAD currency). In particular we were interested in Adjusted Close Prices – close prices adjusted for dividends and splits. This data includes 252 monthly observations that were taken for the period from November 1992 to November 2013 for 10 Canadian stocks.

Each company in our dataset is either well-known across the Canada: Bombardier Inc (C:BBD.B), Bank of Montreal (C:BMO), Toronto Dominion Bank (C:TD), Loblaw(C:L), Canadian Tire (C:CTC.A), Sears Canada (C:SCC), Rogers Communications (C:RCI.B), SNC-Lavalin Group (C:SNC), Canada Bread Co. (C:CBY). Following the findings of Chan et al. (1999), to make variance covariance matrix noisiness lower we considered mainly stocks of big firms (in terms of market capitalization). Also we include well known in Guelph company Linamar (C:LNR) (Table 1).

We divide data into two sub-periods. Data before January 1998 we use just as information for the constructing Min-Var portfolio in January 1998. Period January 1998 – November 2013 is used for constructing, active trading and rebalancing of all portfolios. So, it is important to report actual behavior of the returns for the trading period (Table 2).

As we see returns of some assets dramatically differ in the trading period vs overall sample. For example BBD did very well before trading period causing overall mean return to be 0.45%. However during trading period mean return of BBD was -0.24%.

3 Empirical methods and methodology

3.1. Benchmark portfolio (without reweighting)

To evaluate the performance of minimum variance strategy we used the equal-weight portfolio, in which wealth allocated evenly across the assets in January 1998 and no reweighting is happening.

Another 1/N portfolio that we present (although it is not benchmark in our study), is when every period assets are allocated so that they have 1 over n share in the portfolio every period. It should be noticed that this simple strategy needs to be rebalanced because price changes drives assets weights away from 1/N portfolio.

Both these strategies are very easy to implement as they does not involve any estimation and only impose 1/N weights for second portfolio.

3.2. Maximum Expected return optimization

Formally, Return Maximization strategy can be written as:

Minimize the portfolio variance:

Table 1 Descriptive statistics of monthly returns (November 1992 – November 2013)

	N	*Min*	*Max*	*Range*	*Median*	*Mean*	*Var*	*Std.Dev.*
C:BBD.B	252	-75.68%	37.47%	113.15%	1.15%	0.45%	0.0165	12.85%
C:LNR	252	-57.51%	101.24%	158.75%	1.11%	0.98%	0.0166	12.89%
C:BMO	252	-28.00%	19.95%	47.95%	1.04%	0.73%	0.0043	6.52%
C:TD	252	-32.18%	20.64%	52.82%	1.08%	0.96%	0.0041	6.38%
C:L	252	-26.23%	21.53%	47.76%	0.59%	0.82%	0.0036	5.98%
C:CTC.A	252	-48.06%	18.06%	66.13%	1.31%	0.71%	0.0054	7.33%
C:SCC	252	-57.30%	28.69%	85.99%	1.29%	0.42%	0.0107	10.36%
C:RCI.B	252	-37.01%	36.67%	73.68%	0.89%	0.78%	0.0113	10.61%
C:SNC	252	-37.42%	21.96%	59.38%	1.91%	1.51%	0.0062	7.86%
C:CBY	252	-41.96%	21.71%	63.67%	0.57%	0.62%	0.0067	8.21%
T-bills	252	0.01%	0.69%	0.68%	0.26%	0.27%	0.0000	0.16%

Table 2 Descriptive statistics of monthly returns (January 1998 – November 2013)

	N	*Min*	*Max*	*Range*	*Median*	*Mean*	*Var*	*Std.Dev.*
C:BBD.B	191	-75.68%	37.47%	113.15%	0.00%	-0.24%	0.0200	14.15%
C:LNR	191	-57.51%	101.24%	158.75%	0.26%	0.16%	0.0200	14.13%
C:BMO	191	-28.00%	19.95%	47.95%	1.02%	0.42%	0.0046	6.80%
C:TD	191	-32.18%	20.64%	52.82%	1.07%	0.67%	0.0045	6.72%
C:L	191	-26.23%	20.13%	46.36%	0.04%	0.32%	0.0035	5.95%
C:CTC.A	191	-48.06%	18.06%	66.13%	0.97%	0.61%	0.0053	7.30%
C:SCC	191	-57.30%	28.69%	85.99%	1.38%	-0.12%	0.0118	10.86%
C:RCI.B	191	-37.01%	36.67%	73.68%	1.58%	1.35%	0.0120	10.95%
C:SNC	191	-37.42%	21.91%	59.33%	1.72%	1.32%	0.0060	7.76%
C:CBY	191	-41.96%	21.36%	63.32%	0.60%	0.48%	0.0075	8.67%
T-bills	191	0.01%	0.47%	0.46%	0.22%	0.23%	0.0000	0.14%

$$max_{\omega}\ \omega' m + (1 - \omega * i) * rf.$$

Subject to: $(\sigma^*)^2 = \omega' \Sigma \omega$

Where ω is the vector of portfolio weights that should be implemented, Σ is expected covariance matrix of assets returns for next period rf is risk free rate, and m is vector of expected returns for next period, i is vector of ones. Given a target value of σ*, efficient portfolio characterized by the solution of the problem – vector of weights. This vector of assets weights describes a proportion of the wealth that is invested in specific asset.

$$\omega_p = \frac{\sigma * \Sigma^{-1}(m - i * rf)}{\left[(m - i * rf)' \Sigma^{-1} (m - i * rf) \wedge (\frac{1}{2})\right]}.$$

Dynamic asset allocation means that optimal portfolio weights recalculated every month based on estimated input parameters. In this process we are trying to predict future portfolio performance by analyzing historical (in-sample data). As time progress the previous period out-of-sample data becomes in-sample data which will be used to estimate parameters.

Expected returns are assumed to be formed based on average returns of each asset over last five years for 5 years rolling window portfolio and since November 1992 for recursive portfolio. Similarly covariance matrix is constructed using information over last five years prior to forming weights for 5 years rolling window portfolio; and since November 1992 for recursive portfolio. Annualized Sigmastar for optimization has been chosen equal to 15.7%. This value is not arbitrary. Ex Poste sigma for 1/n portfolio was equal to 15.7%.

3.3. Performance evaluation

The performance evaluation of the minimum variance and benchmark strategies is based on Ex Post Sharpe ratios by methodology taken from W. Sharpe web page (http://www.stanford.edu/~wfsharpe/art/sr/sr.htm) Sharpe ratio measures return per unit of risk, meaning it allows comparing risk adjusted returns of investment strategies.

4 Results

In this section we evaluate the out-of-sample performance of the dynamic minimum variance portfolios and compare them to an equally weighted portfolio, which is commonly considered as a naive benchmark for testing different asset allocation strategies. In addition to Sharpe ratio, we also report variation of the weights over time in minimum variance portfolio and cumulative return graph for both, minimum variance and equally-weighted portfolios.

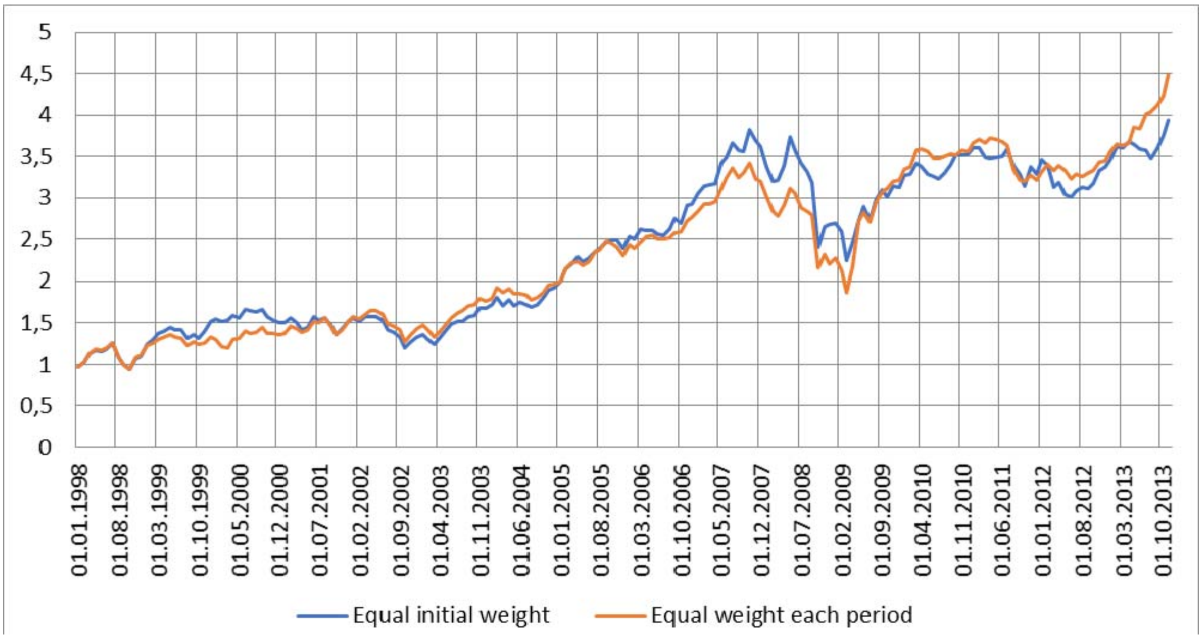
We run two portfolio optimization processes (rolling and recursive estimation windows) both are out of sample, one in sample process, and compare each to benchmark.

Graph 1 shows that both equal initial weight portfolio (without rebalancing) and equal weight portfolio with rebalancing each period have shown upward sloping trend over the last 15 years. Both of them were severally affected by recession in 2008–2009. Also, both of the recovered after and currently are on their historical (for this time frame) maximum. Portfolio with every period rebalancing did better that one shot 1 over N portfolio. Ex Post Sharpe Ratios are 0.103 and 0.119 respectively. There can be several explanations to this fact.

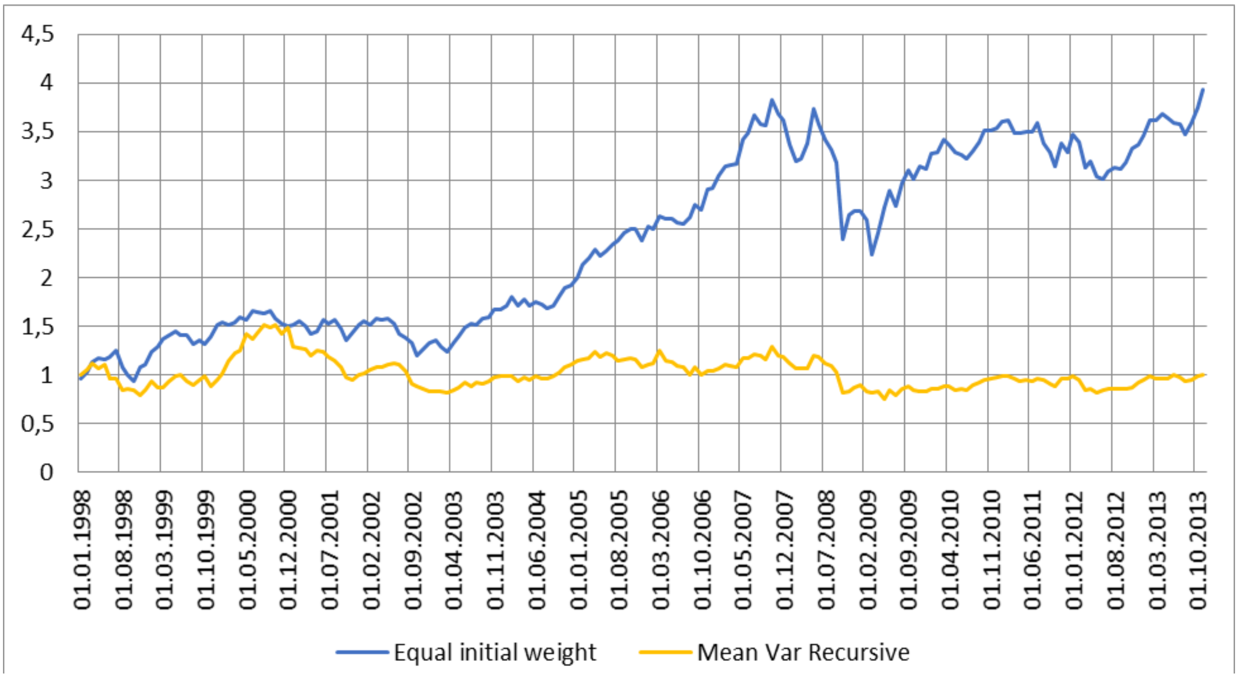
First it can be because of some mean reversion in the data. Portfolio with rebalancing is selling stocks when their price goes up and buys them when their price is going down. If mean reversion is presented this strategy over performs benchmark in terms of return. Average correlation between expected and realized returns is about 0.1. Second, even without mean reversion portfolio that has 1/N weight every period is more diversified. It reduces impact of idiosyncratic risks, causing smaller st.dev. Hence Sharpe Ration will be higher for less volatile portfolio.

Surprisingly Mean-Var portfolio (based on recursive information set) did not perform well. Cumulative return for it over 15 years makes value of this portfolio almost the same as it was 15 years ago, adding only 0.6% in total. This portfolio had showed good results for the first years adding 52% in value, but then it went down (please see Graph 2). There are several possible explanations to this fact.

First of all, as it was discussed in the literature before it is very hard to predict returns for portfolios and it is even harder to do so for individual stocks. Taking average return over past as an expectation of return (and covariance) for the next period might be misleading. When



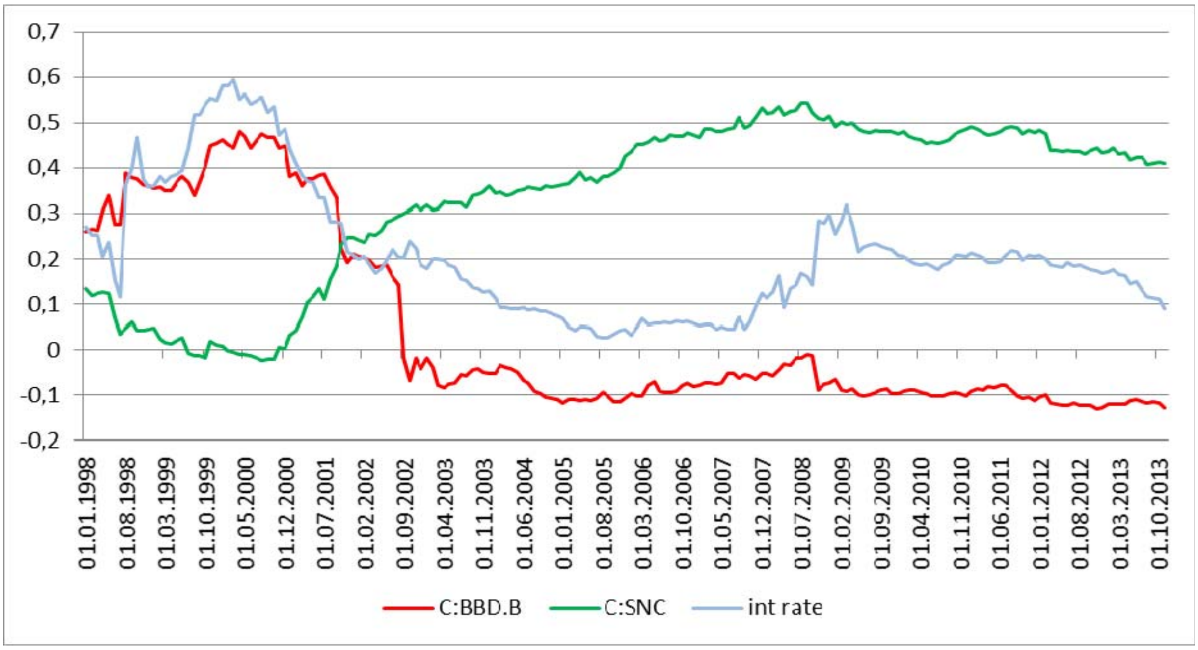
Graph 1 Cumulative return. One over N portfolios with and without rebalancing



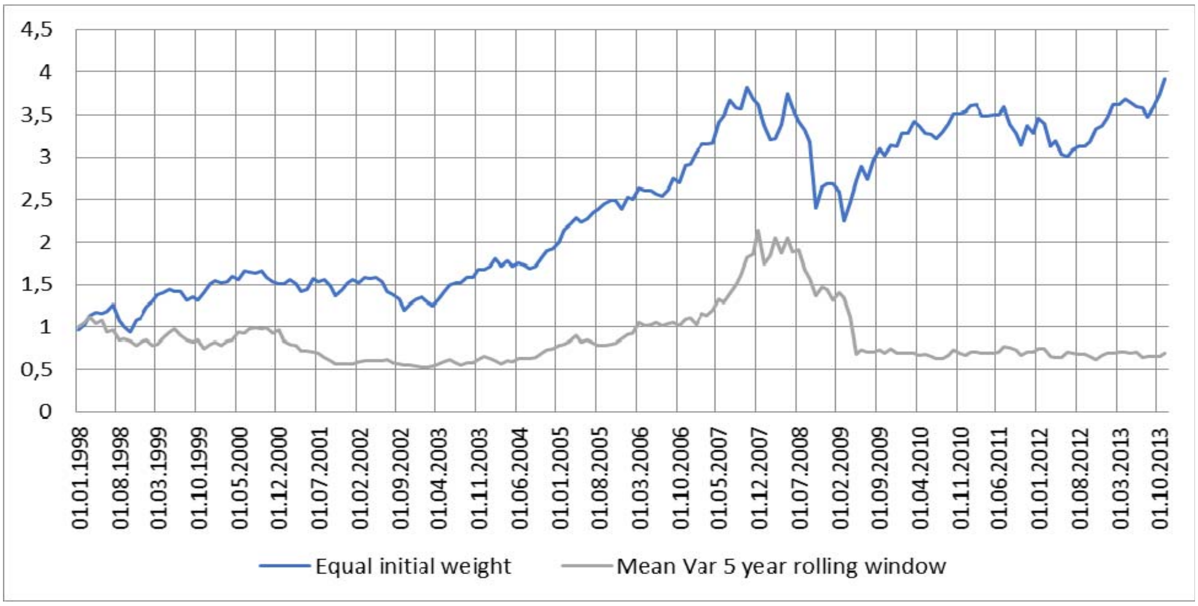
Graph 2 Cumulative return. Benchmark vs Mean-Var portfolio based on recursive information set

stock price goes higher than expected, return exceeds expectations, and then the model adjust expectations a higher levels of return. It makes investor to sell some other stocks that performed not as good and to buy even more stocks with high price. If then stock price underperform, returns are lower than it was expected and investor builds lower expectations. Hence he sells some underperforming asset. Given this approach of building expectations investor will be more likely to buy stocks after their price went up and to sell stock after their price went down. Another possible explanation is that expected covariance is not accurate and distorts weights allocation.

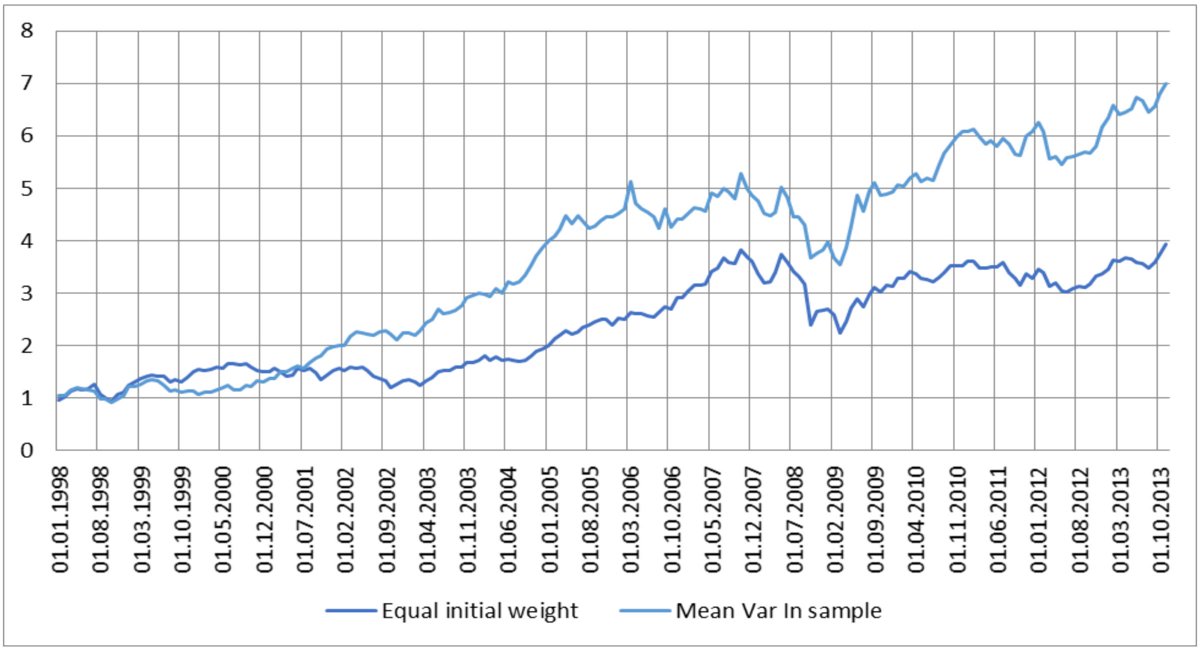
Finally, Graph 3 shows weights of Mean-Var portfolio with recursive information set on example of two risky assets and risk free asset. As we in the beginning of period model suggests to put 30–45% in the Bombardier stocks, while end of period suggests that investor should had gone short 10% on Bombardier stocks. Almost opposite situation is happening with SNC-Lavalin stocks. While initially



Graph 3 Some weights of Mean-Var portfolio with recursive information set



Graph 4 Cumulative return. Benchmark vs Mean-Var portfolio based on 5 years rolling window infoset



Graph 5 Cumulative return. Benchmark vs Mean-Var portfolio based on in sample information set

Graph 6 Cumulative return. One shot Mean-Var portfolio

target weight was varying between zero and 10%, in Ex Poste it suggests that these stocks should have 40–50% of portfolio. As we will discuss in the 5th section even one shot Mean-Var portfolio in the beginning of the sample would give better performance that monthly reallocating of weights based on averaging past returns.

Mean-Var portfolio works perfectly in sample. If investor knew covariance and cumulative return of stock, he would chose in the beginning weights that he assigns in the last date. It is shown on Graph 5 that "in sample" Mean-Var portfolio beats benchmark. It shows that investor is backward rational, but not forward rational.

Graph 4 shows similar situation for Mean-Var portfolio with 5 years rolling window. However 5 years rolling window is more influence but recent situation. Portfolio was underperforming almost all the time. However, this portfolio was doing very well right before the 2008 crisis. It doubled the value within one year prior to the December 2007, while factor of benchmark portfolio was 1.3 for the same period. However during the crisis this portfolio went down and did not recover much since that time.

Ex Poste Sharp ratios of Mean-Var portfolios are negative -0.058, and -0.042 risk free asset bet performance of both Mean-Var portfolios.

5. Roles of transaction costs

Even though in our study we assumed transaction cost as zero, in real market operation they are not zero. Transaction fees paid every time an investor completes a transaction. From the point of view of the study – main factors that influence the size of transaction fees are frequency and size of transaction.

In our empirical study we kept rebalancing every month, so the main factor to consider is the size of transaction. Portfolio rebalancing done once every month according to new optimal weights that produced by specific minimum variance strategy. In other words, to adjust his investment portfolio to the new optimal weights investor required execute trading operations of difference between achieved and desired weights.

One over N portfolio without rebalancing requires very little transaction costs – only on the day of forming portfolio. After this there is no transaction costs associated with benchmark.

We showed an example of weights change in minimum variance strategy with recursive estimation period portfolio. In the example stock of Bombardier go from about 48% of portfolio weight in 2000 all the way to the short sale of 10% as of 2013. Selling of stocks is made below market value. Buying stocks is made above market value. In both cases transaction costs are charged. So, in a rebalancing of portfolio from asset A to asset B, investor would pay two transaction costs: half of the A asset spread to sell an asset, and half of the B asset spread to buy more of asset B.

Apart from transaction costs associated with spread there are some fees of TSX brokers.

In conclusion, if we are to include transaction costs into portfolio optimization, these will diminish the total profit of the portfolios. Taking into account that, equally weighted portfolio expected to have almost zero transaction costs, transaction costs expected to widen even more the difference in performance of evaluated minimum variance strategies and benchmark portfolio. Although transaction costs play an important part in total portfolio returns, their importance depends on specifications of the optimization problem. Possible extension of the current study should include transaction costs. To make a rebalancing profitable, transaction costs should be lower than expected benefit from rebalancing. It reduces the magnitudes of rebalancing trade. Interestingly, despite the fact that transaction costs have generally negative effect on portfolio performance, it can be the case that their introduction may improve performance. As an extreme case let's assume that right after constructing portfolio in the first period transaction costs went so high, that investor decided to rebalance portfolio. In this case Mean-Var portfolio will be the same as Mean-Var portfolio without rebalancing – once it was constructed, investor do not trade.

As we see from the Graph 6 this portfolio would behave better than Mean-Var portfolios with rebalancing. So, abstaining from rebalancing "saved" investor from wrong choices during rebalancing. However, this portfolio still has worse performance than benchmark portfolio. Once again, it shows that weights constricted based on the extrapolated 1992–1998 information are inferior to simple 1/n weights. But here presence of high transaction costs made investor not to rebalance and kept investor from making other mistakes in the rebalancing.

This puzzle should be attributed only to situations with "wrong" expectations and weighting strategies and it is not generally true.

6 Conclusion

This project constructs and estimates performance of mean-variance portfolios vs naïve portfolio for January 1998 to November 2013, using monthly returns for 10 Canadian stocks over the period from November 1992 to November 2013. Cumulative return and ex post Sharpe ratios are used to quantify comparisons.

It was found that portfolios perform in a following descending order: in sample mean variance portfolio, equal weight with monthly reweighting portfolio, equal weight without monthly reweighting portfolio, mean variance out of sample portfolio with recursive information, mean variance out of sample portfolio with 5 years rolling window.

Naïve portfolios have higher Ex Post Sharpe ratios and higher accumulated return. Most likely it is caused by low predictive power of expected return. Correlation between predicted return and realized return is low. Also some stocks changed dramatically their performance over the period. It caused major losses for mean variance portfolios since they were predicting future performance of assets to be on average equal to the past performance of assets.

We assumed that transaction costs are zero. However, if they were not equal to zero it would not cause a big change in our results, because it would affect more mean-var portfolios that naïve portfolio without reweighting. Hence portfolios that were performing not so well, would perform even worse.

Surprisingly under sum restrictions proper implementation of transaction costs would theoretically even improve mean-var portfolio performance in this case. It is so, because if costs are high enough after the 1st period, investor would be less likely to do rebalancing. As it was shown in this case mean var portfolio would become one shot mean var portfolio. And cumulative return of latter one is higher than cumulative return of latter portfolio.

In a nutshell we recommend investors to use one over N strategy. It is easy to implement. It does not have high transaction costs. And for our sample it outperforms more sophisticated strategies.

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