The Research on an Inter-Organizational Network Structure Using Graph Theory Tools

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Abstract. *In the context of the globalisation of society and the rapid spread of information and computer technologies, the achievement of the goals of individual economic structures is achieved through the formation of a new type of organisational resource by introducing the principles of the network effect and the system of network interaction. The network approach is not reduced to a separate disciplinary area, but claims to be universal and independent. to identify unambiguous attributes that collectively define network interaction. The characteristics of networks are legal autonomy of partners, close connection and mutual influence on the basis of partially formalised or trust-based relationships, establishment of medium- and long-term partnerships, decision-making based on a jointly developed system of internal rules and regulations, taking into account the behavioural patterns and interests of network partners. An inter-organisational network is defined by the links between a certain set of independent organisations (network structure) and their interactions within them (network processes). The structure of network ties and positioning in networks are considered as key properties of network agents. The author's approach proposes to focus not on the agents themselves, regardless of their appearance, motives, norms and values that guide their actions, but on the connections between them in the network, which are studied on the basis of graph theory.The article studies a structure of inter-organisational networks and their new structural features on the basis of graph theory mathematical approaches. Extremum parameters of graphs are used to estimate an optimum number of network members. The aim of the investigation is to apply the mathematical tools of a graph theory to study the basic concepts of a network structure, the justification of the optimal number of network participants and the practical implementation of a network concept. On the basis of theoretical studies using graph theory, proposals for the structure of the network can be formulated, which can be used to stimulate sectoral restructuring and change the technological platform of the national economy.*

Keywords: *network economy, inter-organisational network, relationship structure, graphs, centrality, polarity, integration.*

JEL Classification: *C22, L19, L22*

1 Introduction

Modern world changes are grounded on intellectual and informational network capabilities. The post-industrial society is based on network principles and network economy is the result of knowledge and the newest information and communication technologies. In accordance with global trends, the transition from closed economic entities to open-type business systems with balanced partnership relations promotes the activation of business potential, its incorporation into the competitive environment by forming network mechanisms of interaction between economic entities and new organizational forms of entrepreneurial structures arising on their basis.

The number of macro- and micro-economic structures functioning on network principles is constantly increasing. The network effect of interaction generates a new phenomenon of growing marginal utility and increasing marginal productivity. The greater the scale of activity in such conditions is, the greater the efficiency of the use of additionally attracted resources is. The effect of the scale is shown especially brightly within the network which spreads its own standards, which becomes the main factor of competitiveness.

The object of the research is inter-organizational networks (MoS) as an alternative to the hierarchical economic systems of business organization. They are considered through the relationships and

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interrelations between economic agents (network enterprises) and their central characteristic is the structure of relations between the participants. The relations between economic agents in an inter-organizational network will be further treated as institutions with their own structure, which is predominantly stable and predictable in real time. The totality and structure of relations are considered to be an inter-organizational network.

The distribution of a network concept and the corresponding growth of researchers' attention to this subject is caused by the possibility of the use of an interdisciplinary approach.

2 Prerequisites for studying the structure of inter-organisational networks

MoS is a self-regulating system built on the principles of synergy and the reduction of investment risks for optimal use of resources including knowledge and intellectual potential. Sociology started the study of network interaction. American sociologist and social psychologist J. Homans's theory of social exchange was built on the idea of the exchange of social and material resources as a fundamental way of forming interactions in society [1]. M. Granovetter [2] emphasized that economic analysis strengthens the importance of relationships between the participants of a business network due to the introduction of the network into the social structure.

When MoS is being formed, first of all its basis is constructed in the form of a social network as a specific set of connections between a certain group of economic agents. Interdisciplinary theory of social networks has strengthened not only the network concept, but also the role of graph theory tools in a network research which allowed to connect the concept of a graph theory with structural properties of networks [3–5].

The analysis of the main theoretical and empirical research statements in network economy raises the need for theoretical justification of an optimal network structure, the mechanisms of network interaction and the coordination of the actions of individual enterprises-participants, the role of economic players on branch markets and in the competitive environment.

The interpretation of MoS as institution networks identifies them with organizational forms of activity, where the main role is the reduction of uncertainty due to the stable structural interrelations and relationships. The purpose of the research on the representation of MoSs as institutions is to establish norms and rules that order the specific relationships between the supposed participants of a network. The research on the structure and mechanisms of the coordination of network participants' actions allows to analyze a real business process, MoS acts as a tool for the realization of an innovative-investment model.

Most of modern scientific works are devoted to the study of differences between the market, hierarchy and networks, definition of prerequisites for the formation, classification and main characteristics of a network. The mathematical tools of a graph theory, in particular, the property of graph centrality have already been used in the study of social networks.

The authors offer to use additional structural properties of graphs – integration and polarity (unipolarity). These structural properties of graphs have been studied [6–7], but they are used in the study of the structure and organizational forms of interorganizational networks for the first time.

3 Study of the organisational structure of the network and interconnections of participants based on graph

The study offers to apply the basic terms and tools of a graph theory to construct the basis for an inter-organizational network structure. In practical applications the graph $G=(M, E)$, where the edge $<x, y>$ connects the vertices x and y, is interpreted as a network.

The set of economic agents are elements of the set of graph vertices M, the set of all edges are denoted by E. Studying MoS as a complex system represented as a graph where vertices correspond to system elements and edges correspond to relations between them can be reduced to the task of mathematical classification, the essence of which consists in defining the optimal partition of the set of graph vertices into non-intersecting classes. When algorithmicizing problems and solving them with the help of computer programs, graphs are represented by an incidence matrix.

The distance between vertices is the minimum number of edges that can be moved along to get from one vertex to another. The greatest distance for a given graph is called diameter, and if it is finite, the graph is connected; otherwise, it is incoherent. In the case of a binomial model, it is proved that for sufficiently large n almost all graphs have diameter 2.

A vertex i of a graph is called central if the distance from it to the other vertices is small. The centrality parameter of vertex i is to measure the centrality of distance vector $(d_i 1, d_i 2, ..., d_i n)$, $i = (\overline{1, n})$.

A graph is called integrated if all its vertices are central. Therefore the integration parameter must characterize the degree of centrality on the set

of vertex centrality. A graph is called unipolar if there exists a single central vertex. The unipolarity parameter must coincide with the measure of centrality of the most central vertex, and the center of a graph can be defined as the set of all vertices with minimal sums of distances. A graph is called centralized if the centrality of some vertices strongly differs from the others. The centrality parameter *H* estimates the variance on the set of vertices' centrality.

The relation between the centrality parameter *H*, integration *S*, centrality S_i and unipolarity *V* is represented with the help of the formula:

$$H = 2S - nV = \sum_i (S_i - \min_i S_i), \qquad (1)$$

where $S = \frac{1}{2}\sum_{i,j} d_{ij} = \frac{1}{2}\sum_i S_i$;

$S_i = \sum_j d_{ij}$; $V = \min_i S_i$), $i = \overline{1,n}, j = \overline{1,k}.$

To investigate the structure of a network based on the class of graphs we select for comparison the classes 2-5 which can be used to analyze the vertices of the graph, the connections of vertices with edges and a graph as a whole.

1. The class of vertices of a given graph *G*.
2. The class of graphs with *n* vertices *G(n)*.
3. The class of graphs with *n* vertices and *k* edges *G(n, k)*.
4. The class of connected graphs with *n* vertices *C(n)*.
5. The class of connected graphs with *n* vertices and *k* edges *C(n, k)*.

The use of extreme properties of graph parameters will make it possible to establish the optimal structure of the network; to evaluate the efficiency of the introduction of new agents into the network; to take into account the existing links in the future design; to determine which agents are central and which are peripheral.

When investigating the dependence of the extreme properties of the graph parameters and its structure, a "chain", "star", "cyclic" or "complete graph" are used. The calculation of maximum and minimum values of graph structural parameters is not always possible, and when it is possible, it is reasonable to group those values. Extreme values of parameters at linear normalization for graphs with n vertices are achieved in the case of a "star" and "complete graph", corresponding to the variant of a network with a focal agent and the variant of a polycentric dynamic network with "dyads" connections.

A connected graph has at least *(n-1)* an edge, so for classes *C(n, k)*, if we add edges to the "star", we obtain a graph in which all distances equal 1 or 2 for any of the comparison classes *G(n, k)* and *C(n, k), k ≥ n-1*. In these graphs, the integration *S* takes the minimum value, and the integration calculated on the basis of the sums of inverse distances of a graph takes the maximum value. These values are achieved in the case of a "star", "chain" or "full graph".

Due to the properties of the structural parameters of graphs it is possible to formulate the statements, which should become the theoretical basis for the formation of the organizational structure of a network and the interrelations of its participants.

Statement 1. The structural organization of a network with *n* economic agents and possible *k* links between them determines the maximum value of the sum of density of links between any agent and other ones with only these defined parameters:

$$S \geq C, \text{where} \qquad \mathrm{C} = \mathrm{C}_{\mathrm{n}}^2 + (n-k) - 1, \mathrm{C}_{\mathrm{n}}^2 = \frac{n!}{2!(n-2)!} \cdot \qquad (2)$$

Statement 2. If MoS is formed of five participants (the boundary number) and the maximum value of links with other participants is *k = 4*, then one of the participants becomes central (the property of focality) and links are established between all the other members of the network.

Statement 3. If MoS is formed under the condition $n \geq 6$, and the diameter of the density of connections is 2, then the selected member of the network as a central one establishes interconnections between all the other economic agents.

The maximal value of centrality *H* depends on integration and unipolarity and is reached at minimal unipolarity in the class of connected graphs with *n* vertices and *k* edges (arcs-branches).

Statement 4. If MoS is reproduced by a connected graph and unites at least six participants with established k links, then the property of maximum centralization defines the network structure as a symmetric or maximally approximated to a symmetric tree with three branches.

Possible extreme values of the main parameters of an inter-organizational network are presented in table 1.

The analysis of the data in table 1 shows that the stable communication in three main information-communication channels is established if there are six and more partners in MoS. The process of the network functioning has an evolution character, therefore, according to statements 2 and 3, the number of organizations in MoS must be no less than 6 and this number may increase and the structures a "star" and "full graph" are the most important among the possible structural formations of the interorganizational interaction.

Table 1 Possible extreme values of the main network parameters

Vertices of the graph, *n*	**Minimum value of edges,** *k*	**Maximum value of edges,** *k*	**Maximum value of centralization** *H* **in class** *C(n, k)*	**Minimum value of centralization** *H* **in class** *C(n, k)*
5	4	6	12 (k = 4)	10
6	6	10	20 *(k = 3)*	16
7	8	15	33*(k = 3)*	28
8	10	21	48 *(k = 3)*	39
9	12	28	69 *(k = 3)*	56
10	14	36	96 *(k = 3)*	7

Source: developed by the author

4 Conclusions

In the process of the evolutionary functioning the polycentric network with the structure in the form of a complete graph can transform into the structure with a distinguished central vertex (organization), i.e. structurally it is a "star". However, this process is also possible in the opposite direction. The process of singling out the center or several centers does not mean the construction of a vertical hierarchy at all, but only the allocation of the organizations with higher innovative, creative, financial capabilities in the structure of the network.

The analysis of published results of empirical studies shows that polycentricity of MoS is not always stable. In a network the center is allocated conditionally, that center assumes functions of a coordinator as a managing center. Such evolution of MoS is reduced to displacement of forces in correlation with greater resource opportunities, with emergence of forces with greater market power and changes in the structure and organizational form of a network. The tasks of coordination and project management are often delegated to a special center, a coordinator, rather than to individual network partners.

Such center, as an agent-coordinator, allocates partners with the best competences for specific problems and tasks, coordinates conflicts as a result of dynamic competition between economic agents competing for participation in projects. The decision of distribution of competences between partners by means of outsourcing will allow to concentrate attention on the development of innovative projects, products and activities, which is key accordingly to the strategic goal of a network.

The study of a network concept will allow Ukraine to create new economic and technological realities which should be implemented in the innovation strategy of national development. A new global challenge is theoretically formed which stimulates sectoral restructuring and changes in the technological platform of the national economy in its transition to the innovative model of a structural adjustment.

The development strategy for Ukraine before a full-scale war was based on the development of national and regional innovation systems, technological foresight, activation of private partnership in the commercialization of technology and formation of investment demand for innovation. But, in the practical aspect, the mentioned provisions have not found their implementation, even in the formation of infrastructure, where only certain types of innovative structures are represented, in particular, technoparks, research centers, business incubators, scientific and technical enterprises, foundations. And the activity of only a small part meets the tasks that should be solved by them, based on the world experience of the organization of various types of innovative structures. At the same time the need for the implementation of the concept of a "technological breakthrough" based on the formation of new forms of local formations – clusters, special economic zones, technopolises – and building a new network form of economy has matured.

Ukraine cannot ignore the strategic direction of the economic development as it will further consolidate the status of a country-outsider with a range of negative consequences. That is why the state should work out a strategy taking into account the protection of national interests, acquisition of new key technologies and conquering the atypical segments of the world market. With the priority support of advanced technologies and sectors as well as the creation of a new industrial infrastructure, the most important direction of the innovation policy should become the development of network cooperation and networking.

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